Floyd R. Newman
Laboratory of Nuclear Studies
Cornell University
Newman Lab. Cornell Univ.
Ithaca, New York 14853
607-256- 4397

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Dr. J. W. Harris
Lawrence Berkeley Laboratory , 50D-115
University of California
Berkeley, CA 94720

Dear Dr. Harris:

I have studied your paper LBL-19114 in some detail. I think it is a very important experiment, and will be useful for our theory of supernova explosions. There are, however, some questions.

1) I cannot reproduce your formula (18) from the paper by Chapline et al., your ref. 8. I assume that the Chapline theory is correct. On the enclosed handwritten notes, I transform Chapline's eq. (6) into a form which I find more convenient, my eq. (5). This equation I like because in the non-relativistic limit, $w \ll M$, it goes over into the well-known Hugoniot equation. So I used my (5).

On the other hand, I tried to compare your (18) with Chapline's (6), and found a substantial discrepancy. This is also in the handwritten notes.

2) I presume the calculation of thermal energy and pressure from the observed number of π is straightforward. Nevertheless, I should like to get your numbers for the temperature and thermal pressure as a function of $E_{c.m.}$, using the chemical model. I presume you measure the total number of π, i.e. $\pi^+ + \pi^-$ (and possibly also π^0), and not just π^-, as Fig. 5 might lead you to expect. My estimate is that at the highest compression, the temperature T = 100 MeV, and that you should add to the pressure about 9% for the π mesons. But this estimate is very rough.

I expect you separate Δ from π at maximum density by using thermal equilibrium.

3) To use my eq. (5), it is necessary to know $P/\rho w_c$ in the compressed state. Here w is directly measured. P/ρ could be obtained by differentiating w, but this is a very inaccurate method like any differentiation of empirical data. I therefore prefer to assume an analytical formula for w, and differentiating that. I found the following to be a good approximation

$$w_c = K(u-1)^2 \qquad (6)$$

where $u = \rho/\rho_0$. This formula also has the correct behavior at $u = 1$. From this, I obtain

$$P_c/\rho = w_c\, 2\frac{u}{u-1} \qquad (7)$$

Then one merely needs a rough estimate of u to obtain the value of P_c, the pressure due to compression. The results are again given in the handwritten notes.

From these notes you see that at your highest compression, $u = \rho/\rho_0$ is 3.6 while your value is 3.8. The difference is not great.

Could you please go through these arguments and write me whether my evaluation is correct, or if not why not.

4) By the way, your E_c and my w are not the compressed energies at T = 0. I accept your separation of energy ε into a thermal and a compressional part. However, at T = 0 has to be added the Fermi energy of the nucleons to E_c in order to get the total energy. At high temperatures like yours, the Fermi energy is replaced by the usual thermal energy. The Fermi energy is of the order of 20 MeV, not negligible.

5) Your experiments were chiefly on Ar + KCl. These are not very heavy nuclei. So I like your experiment on La + La much better. Have you got any more definite results from this? Is there a chance that you can go to U + U?

Once more, thank you for sending me your papers.

Yours sincerely,

Hans Bethe

Hans A. Bethe

HAB:vhr

Enclosure

To Dr. John Harris

1. Compression ρ/ρ_0.

Start from Chapline et al, Phys Rev D 8 (1973), 4302.

Eq. (6) says

$$\frac{\rho}{\rho_0} = \gamma_{c.m.} + \frac{n E}{2P} \qquad (n = \rho) \qquad (1)$$

$E = E_{lab}$ = ~~kinetic energy~~ in lab, per baryon. Now

$$\gamma_{c.m.} = \sqrt{1 + \frac{E}{2M}} \qquad (2)$$

$$E = 2M(\gamma_{c.m.}^2 - 1)$$

Write the energy per particle in c.m.

$$\varepsilon = M\gamma_{c.m.} = M + w. \qquad (3)$$

w is essentially your W in eq. (17); it excludes M. After a little algebra,

$$E = 2w\left(2 + \frac{w}{M}\right) \qquad (4)$$

Inserting into (1),

$$\frac{\rho}{\rho_0} = 1 + \frac{w}{M} + \frac{\rho w}{P}\left(2 + \frac{w}{M}\right) \qquad (5)$$

$$= \left(1 + 2\frac{\rho w}{P}\right)\left(1 + \frac{w}{2M}\right) + \frac{w}{2M}$$

Concerning your eq. (18), I don't think it agrees with Chapline's, (1). We have the question

$$\frac{\gamma}{1 - \rho_0 \frac{E_{lab}}{2P}} \stackrel{?}{=} \gamma + \frac{nE}{2P} \qquad (A)$$

Multiply by denom. on left hand side gives (note $n = \rho_0$)

$$\gamma \overset{?}{=} \gamma - (\gamma - 1)\rho_0 \frac{E_{lab}}{2P} - \rho_0^2 \left(\frac{E}{2P}\right)^2 \qquad (B)$$

This cannot be true because third and ~~fourth~~ second term on right hand side are both negative.

2. Thermal energy and pressure.

I presume the evaluation of these quantities from the observed $n_\pi + n_\Delta$ is straightforward

3. Thermal pressure (at highest incident energy)

$$P_t/\rho \simeq T\left(1 + \frac{n_\pi}{A}\right) \simeq 100(1 + .09) = 109 \text{ MeV} \qquad (8).$$

Compressional pressure

$$P_c/\rho \simeq w_c \, 2 \frac{u}{u-1} = 115 \cdot 2 \cdot \frac{4.4}{3.4} = 296 \text{ MeV} \qquad (9)$$

$$P/\rho = 405 \text{ MeV} \qquad (10)$$

$$w = 375 \text{ MeV (measured)} \qquad (11).$$

$$\frac{2w\rho}{P} = 1.85$$

$$w/M = .40.$$

$$\rho/\rho_0 = 3.63$$

H. A. BETHE
209 WHITE PARK ROAD
ITHACA, NY 14850

Dr. John Harris
Lawrence Berkeley Lab., 500-115
1 Cyclotron Rd.
Berkeley, CA 94720

Bethe
NEWMAN LABORATORY
Cornell University
Ithaca, N. Y. 14853

Dr. J. W. Harris
Lawrence Berkeley Laboratory, 50D-115
University of California
Berkeley, CA 94720